Destruction of global coherence in long superconducting nanocylinders

V. H. Dao and L. F. Chibotaru
Institute for Nanoscale Physics and Chemistry (INPAC) and Division of Quantum and Physical Chemistry, University of Leuven, Celestijnenlaan 200F, B-3001 Leuven, Belgium

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Recent experiments on long hollow nanocylinders have reported an anomalous broadening and a multistep character of the resistive transition between normal and superconducting states with increasing magnetic field. Here we show that a first-order phase separation is not the reason for such behavior since it can arise only for a wall thickness that is much larger than in the investigated samples (~30 nm). On the contrary, it is found that the destruction of global coherence in the transition neighborhood should be triggered by intrinsic long-range inhomogeneities of the cylinder.

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I. INTRODUCTION

Beside its obvious benefits to technological applications, the physics of charge transport in mesoscopic structures represents a paradigm for phase transition in low-dimension systems, and especially quantum phase transition,[1–3] i.e., a transition that occurs at zero temperature by changing another parameter such as the applied magnetic field, the pressure, or the level of disorder. Among the most investigated but still unsolved problem is the insulator-superconductor transition.[3] Its understanding can play a key role for explaining superconductivity in cuprates. However even in the case of the metal-superconductor transition with standard BCS mechanism, the effects of quantum fluctuations are debated.[2–5] One possible version of this phenomenon is the destructive regime of superconducting hollow nanocylinders. When the radius R of the tube is smaller than the zero-temperature coherence length $\xi(0)$, the kinetic energy of the supercurrent exceeds the condensation energy around half-flux quanta of applied magnetic flux, $\Phi=(n+1/2)\Phi_0$, and the periodic destruction of superconductivity at zero temperature occurs.[6] Recent realizations[7,8] of the Little-Parks experiment[9] with ultrathin aluminum tubes confirmed this prediction, however they also reported a multistep resistive transition with a temperature width that unexpectedly broadens when departing from the zero-field critical temperature $T_c(0)$.

To explain these observations, previous theoretical investigations[4,5] focused on the modification of the conductivity by critical fluctuations[10] in a one-dimensional system but their conclusions diverge. Vafek *et al.*[4] proposed that quantum critical fluctuations are responsible for the anomalous resistance while thermal fluctuations have to be disregarded for their effects are several orders of magnitude too small. However their calculations are limited to high temperatures. On the other hand, Shah and Lopatin[5] derived a diagrammatic formalism valid to all temperatures and concluded that quantum fluctuations are important only in a narrow vicinity of $T=0$ at the border of the parameter range over which the experiments were carried on. But they found qualitative behaviors of the resistance that were not experimentally observed: a narrowing of the resistance drop for increasing field in the classical regime and a nonmonotonous temperature dependence in the quantum regime. Finally none of the cited works have quantitatively reproduced experimental results and, first of all, the steplike features observed in the temperature variations in the resistance.

The discrepancies between the above theories and the experiments lead to question the assumptions adopted by these models, namely, the role played by fluctuations, the nature of the disorder, and the effective dimension of the system. On the other hand, the mean-field approximation proved to be quite successful in describing low-T_c superconductors, even for mesoscopic samples in cases when the order parameter varies on length scales larger than the coherence length,[11] because the Ginzburg parameter (i.e., the ratio of the thermal energy to the characteristic fluctuation mode energy) is generally very small. Therefore it is reasonable to begin with it before considering the effects of fluctuations. So in the present paper we investigate possible mean-field explanations of the observed phenomena. The reported resistive behavior during the transition suggests that the puzzling conduction properties result from a separation of the tube into normal and superconducting sections since the resistance is always lower than the normal one. We find that the transition is of second order for the studied cylinders so that the phase separation must be triggered by the tube inhomogeneities. We then propose a minimal model where structural properties, i.e., geometric or normal-state parameters such as the electron diffusivity, vary along the cylinder. The originality of our model compared to previous ones is that the disorder is modulated at the long-range scale: the parameters are assumed to change smoothly. This yields an effective spatial distribution of $\xi(0)$ and of the local mean-field transition temperature $T_c(H)$ which explains the destruction of global coherence and the peculiar variations in the resistance. The paper gives the detailed description and an extension of the results in a previously published comment.[12] Its plan is the following: in Sec. II we investigate the phase transition in a perfectly homogeneous cylinder within the Ginzburg-Landau (GL) theory; a minimal BCS model with inhomogeneous disorder is proposed in Sec. III and the calculated resistance in the limit of vanishing current is compared to experimental data in Sec. IV; Sec. V discusses on the model validity as well as possible extensions, and it includes the conclusion.

II. GL THEORY OF A PERFECT CYLINDER

In order to interpret their resistance curves Wang *et al.*[8] proposed a scenario in which the superconducting phase is regularly split into 2^n identical regions separated by normal interfaces and they suggested that such a process is induced by quantum fluctuations. Although an inhomogeneous phase due to fluctuations was predicted close to the quantum critical point by theory,[13] it was also shown that only a glassy phase can arise, not a regular pattern of alternating normal and superconducting regions. There are no clear physical reasons why the proposed spontaneous separation in regular segments should be stabilized by quantum fluctuations nor why the latter should be effective at lower fields and finite temperatures. Thus we first check within a mean-field approach, the GL theory,[14–16] whether an inhomogeneous phase can occur in a perfectly homogeneous cylinder.

The equilibrium state is found by minimizing the GL functional F with respect to the complex superconducting order parameter Ψ and the local magnetic induction $\mathbf{b}=\nabla\times\mathbf{A}$. We consider here only cylindrical symmetric solutions, having in mind tubes with a radius of the order of the coherence length where a gradient of the order parameter amplitude in the orthoradial direction would cost too much energy. We modify the standard form

$$F=\int \alpha|\Psi|^2+\frac{\beta}{2}|\Psi|^4+K|\Pi\Psi|^2+\frac{1}{8\pi}(\mathbf{b}-\mathbf{H})^2, \qquad (1)$$

where $\Pi=-i\nabla+\frac{2\pi}{\Phi_0}\mathbf{A}$ and the uniform external magnetic field $\mathbf{H}=H\mathbf{e}_z$ is applied along the cylinder by introducing $\Psi=\sqrt{|\alpha|/\beta}fe^{i\chi}$ (f and χ are real), the coherence length $\xi=\sqrt{K/|\alpha|}$, and the magnetic penetration depth $\lambda=\sqrt{\Phi_0^2\beta/32\pi^3|\alpha|K}$. Then minimizing with respect to the vector potential yields the Maxwell equation,

$$\nabla\times\nabla\times\mathbf{A}=\frac{4\pi}{c}\mathbf{j}_s=-\lambda^{-2}\left(\frac{\Phi_0}{2\pi}\nabla\chi+\mathbf{A}\right)f^2, \qquad (2)$$

which enables us to write the normalized functional

$$\tilde{F}=\int dV\left[(\xi^2\mathbf{Q}^2-1)f^2+\frac{1}{2}f^4+\frac{2\pi\xi^2}{\Phi_0}\mathbf{A}_i\mathbf{Q}f^2+\xi^2|\nabla f|^2\right] \qquad (3)$$

as depending only on the order parameter. Here $\mathbf{Q}=\nabla\chi+\frac{2\pi}{\Phi_0}\mathbf{A}_\mathbf{H}$ with the applied vector potential $\mathbf{A}_\mathbf{H}=\frac{Hr}{2}\mathbf{e}_\theta$, and induced $\mathbf{A}_i(\mathbf{r})=\frac{1}{c}\int dr'^3\frac{\mathbf{j}_s(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}$ is expressed as

$$\mathbf{A}_i(\mathbf{r})=\frac{\Phi_0}{2\pi}\sum_{n=1}^{\infty}\int dr_1^3W(\mathbf{r},\mathbf{r}_1)\cdots\int dr_n^3W(\mathbf{r}_{n-1},\mathbf{r}_n)\mathbf{Q}(\mathbf{r}_n) \qquad (4)$$

with $W(\mathbf{r},\mathbf{r}')=-f^2(\mathbf{r}')/4\pi\lambda^2|\mathbf{r}-\mathbf{r}'|$.

We now derive a criterion to determine the order of the phase transition.[17] The equilibrium energy at finite field is perturbatively calculated from the zero-field ground state which undergoes a second-order transition. In this case superconductivity continuously vanishes at the transition and is uniform along the axial direction. In the limit of small cylinder-wall thickness $t\ll\xi$ and for applied flux $|\Phi=\pi R^2H|\lesssim\Phi_0$, the distribution of the order parameter can also be considered uniform in the radial direction so that $\Psi(\mathbf{r})=fe^{in\theta}$ (n is integer) to a good approximation.[18] f is actually the mean value of the exact amplitude. The gradient term $|\nabla f|^2$ in functional (3) can then be neglected and keeping only the first contribution in expansion (4) of $\mathbf{A}_i$ is enough. Adding the latter to $f^4/2$ yields the coefficient

$$c_4=\frac{1}{2}-\frac{\xi^2}{4\pi\lambda^2\mathcal{V}}\int dr^3\int dr'^3\frac{\mathbf{Q}(\mathbf{r})\mathbf{Q}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} \qquad (5)$$

for the term of order f^4 in the functional. Here $\mathcal{V}=2\pi RtL$ is the volume of the wall and L is the length of the cylinder. The normal-state/superconductor transition can turn into a first-order one by changing the applied field if c_4 becomes negative. Indeed the dependence $\tilde{F}(f)$ can then be nonmonotonous and we obtain a metastable state at a finite value of f stabilized by the higher-order powers of f in decomposition (4). At higher field values this state can eventually become the ground state, leading to a first-order transition.[15] In the limit $R\ll L$, the condition $c_4<0$ yields the simple expression

$$\left(\frac{\lambda}{\xi}\right)^2<\left(\frac{\Phi}{\Phi_0}\right)^2 g_{t,R} \qquad (6)$$

for $|\Phi|\le\Phi_0/2$ (as superconductivity nucleates with vorticity number $n=0$) where the geometrical factor $g_{t,R}\approx t/R$, e.g., $g_{t,R}\approx 0.36$ for $t/R=0.4$ in the experiments,[7,8] and more generally when $t\ll R$,

$$\left(\frac{\lambda}{\xi}\right)^2<\left(n+\frac{\Phi}{\Phi_0}\right)^2\frac{t}{R}. \qquad (7)$$

So for well chosen parameters the same sample can undergo a first-order transition at high field while the order of the transition is two-near zero field. Note also that according to expression (3) the second-order phase transition takes place when $(\xi^2\mathbf{Q}^2-1)=0$ that can be written as $(n+\Phi/\Phi_0)^2=(R/\xi)^2$, so condition (7) takes the form

$$\lambda<\sqrt{Rt}, \qquad (8)$$

which can be compared to the condition $\lambda<t/\sqrt{5}$ for a thin flat film.[15]

We remind that the derived criterion formulated by Eq. (6) and (7), or (8) is a necessary requirement but may not be enough for the emergence of a first-order transition. Nevertheless it is all we need for our present purpose. Using the dirty-limit formula $\kappa=\lambda/\xi=0.715\lambda_L(0)/\ell_{\mathrm{el}}$ with the zero-temperature London penetration depth $\lambda_L(0)\approx 16$ nm for aluminum,[19] we deduce from inequality (7) that the transition cannot be of first order for the mesoscopic cylinders with thickness $t\approx 0.4R$ and electron mean-free path $\ell_{\mathrm{el}}\le 16$ nm studied in experiment.[7,8] The analysis is furthermore supported by the report of no hysteresis. So the possibility of different parts of the tube being in a metastable overcooled or overheated state must be excluded and cannot explain the finite fraction of normal resistance. Thus a perfectly homogeneous cylinder cannot produce the reported anomalous resistance in the mean-field scenario.

III. MINIMAL BCS MODEL WITH DISORDER

Could the sample inhomogeneities then be responsible for the observed phenomena at the transition? For example, the GL expression of the critical temperature at a second-order phase transition for a perfect and infinitely long tube is

$$T_c(H) = T_c(0)\left\{1 - \left[\frac{\xi(0)}{R}\right]^2\left(\frac{\Phi}{\Phi_0}\right)^2\right\} \tag{9}$$

when $n=0$.[6,9] It shows that $T_c(H)$ at finite field changes dramatically with the radius of the cylinder and the coherence length at zero temperature $\xi(0)$. Expression (9) is quantitatively valid only in the vicinity of zero field. The exact $T_c(H)$ is calculated within the microscopic BCS theory.[15,16] The transition temperature is defined by the condition that the linearized self-consistent equation of the gap

$$\ln[T/T_c(0)]\Delta(\mathbf{r}) = [\mathcal{K}(\Pi^2) - \mathcal{K}(0)]\Delta(\mathbf{r}) \tag{10}$$

possesses a nonzero solution.[16] In the dirty limit the operator

$$\mathcal{K}(\hat{q}^2) = 2\pi k_B T \sum_{m=-\infty}^{+\infty} \frac{1}{2|\omega_m| + \frac{1}{3}\hbar v_F \ell_{\mathrm{el}}\hat{q}^2} \tag{11}$$

for $\hat{q}=\Pi$ or 0, where v_F is the Fermi velocity and $\omega_m=(2m+1)\pi k_B T$ are the fermionic Matsubara frequencies. For $t \ll \xi$ the gap function $\Delta(\mathbf{r}) \approx e^{-in\theta}\Delta$ is nearly uniform in the radial direction so we can use the approximation $\Pi^2\Delta(\mathbf{r}) \approx q_n^2\Delta$,[5,18,20] with

$$q_n^2 = \frac{\int (n/r - \pi H r/\Phi_0)^2 r dr}{\int r dr}. \tag{12}$$

The nucleation temperature for $e^{-in\theta}\Delta$ is then the solution of

$$\ln(\tau_n) + \psi\left(\frac{1}{2} + \frac{a_n}{\tau_n}\right) = \psi\left(\frac{1}{2}\right), \tag{13}$$

where ψ is the digamma function, $\tau_n = T_n(H)/T_c(0)$, and

$$a_n = \frac{2}{\pi^2}\frac{\xi(0)^2}{R^2}\left\{(\tilde{\Phi} - n)^2 + \frac{\tilde{t}^2\tilde{\Phi}^2}{4} + n^2\left[\frac{1}{\tilde{t}}\ln\left(\frac{2+\tilde{t}}{2-\tilde{t}}\right) - 1\right]\right\}. \tag{14}$$

Here $\tilde{\Phi} = \pi R^2 H/\Phi_0$, $\tilde{t} = t/R$, and we have used the dirty-limit equality $\xi(0) = \sqrt{\pi\hbar v_F \ell_{\mathrm{el}}/24 k_B T_c(0)}$. The transition temperature $T_c(H)$ is the maximum of the solutions $T_n(H)$. Expressions (13) and (14) indicate that $T_c(H)$ strongly depends on the sample parameters through a_n like, for example, $a_0 \propto (\xi(0) R H)^2$. The explanation of the inhomogeneous transition is then straightforward if the cylinder properties are assumed to vary along the axis and transitions occur locally at different critical temperatures defined by Eq. (13) (see Fig. 1). The finite thickness of the cylinder wall mainly results in a quadratic deviation that is superimposed on the periodic dependence of $T_c(H)$ and vanishes in the limit $t \ll R$ (Refs. 18 and 20) [see Fig. 1(c)].

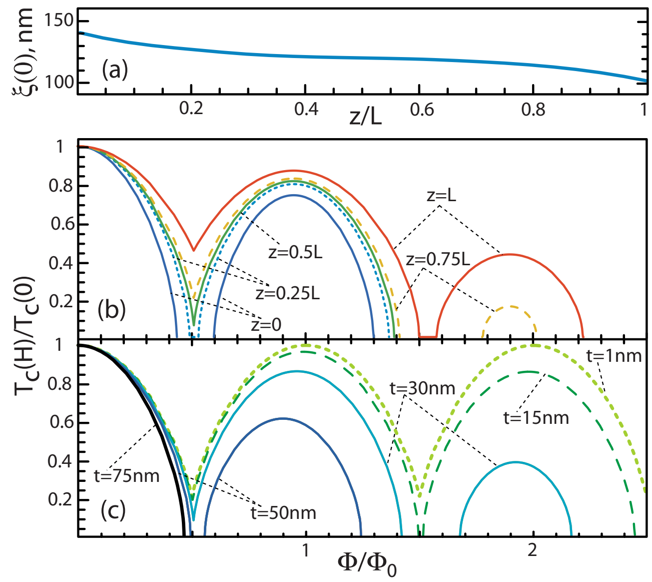

FIG. 1. (Color online) (a) Distribution of $\xi(0)$ along an inhomogeneous cylinder with a constant radius $R=75$ nm. Corresponding local critical temperatures $T_c(H)$ (b) at different positions along the cylinder for a constant wall thickness $t=35$ nm and (c) at position $z=0.5L$ for different values of wall thickness.

The variations in the structural features along the tube axis are actually unavoidable for samples such as the ultralong and thin ones. To calculate their influences on the conduction at finite field, we then propose the minimal model which assumes that the changes are small, continuous, and taking place on a length scale larger than $\xi(0)$. We stress that $T_c(0)$ is the same over the whole cylinder so the zero-field transition is sharp. An approximate invariance of $T_c(0)$ with the variation in the concentration of defects is expected according to Anderson's theorem.[21,22] The hypothesis of small variations for R and t relies on the quality of the samples and seems experimentally fulfilled. The electron diffusivity $D = v_F \ell_{\mathrm{el}}/3$ can change due to an inhomogeneous distribution of crystal defects in the sample and of surface roughness. The effective smoothness of its variations stems from its average influence on superconductivity over a length scale of $\xi(0)$. Furthermore the typical modulation distance is assumed much larger than the coherence length so that proximity effects between normal and superconducting parts are neglected at first approximation. It will be shown below that the assumption of smooth variation is consistent with the experiments. Our theory is locally mean field in the sense that the state of a section at position z along the cylinder is determined by the local transition temperature derived from Eq. (13) with z-dependent parameters. The magnetic response via the screening current of superconducting parts is neglected in the neighborhood of the second-order phase transition. The total resistance of the cylinder is then calculated for small currents as the sum of the resistances of all normal sections with $T_c(H,z) < T$. Note that this simplified model is not suitable to describe the voltage for arbitrary high intensity and the claimed changes of resistance curves at different biases.[8,23]

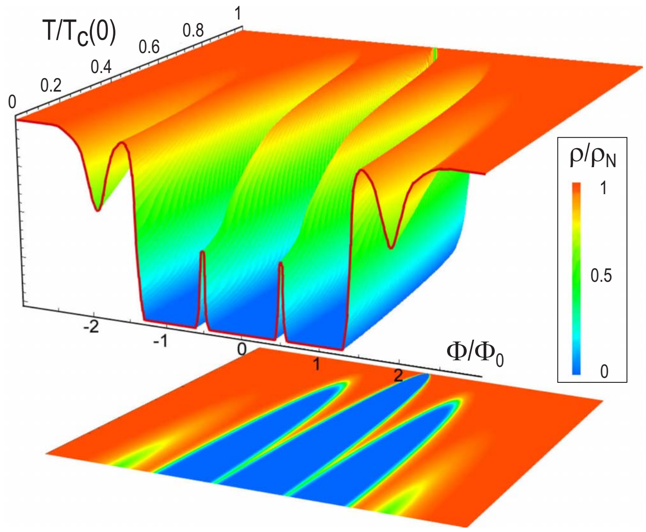

FIG. 2. (Color online) Ratio of the resistance ρ to the normal resistance ρ_N as a function of the applied magnetic field and the temperature for the cylinder with the distribution of $\xi(0)$ plotted in Fig. 1(a).

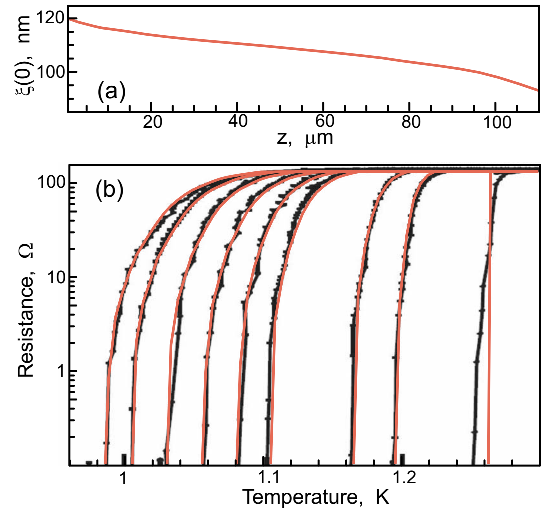

FIG. 3. (Color online) (a) Proposal for a monotonous $\xi(0)$ distribution along the cylinder and (b) the corresponding temperature dependences of the resistance (lines) at different field values (from left to right, H=190, 184, 175, 165, 155, 145, 115, 95, and 0 G), which fit the experimental data (joined dots) of sample Al-6 from Ref. 8.

IV. RESISTANCE VS H AND T

A. Broadening of the resistive transition

The broadening of the temperature width of the transition at finite magnetic field can be understood within our model as due to the inhomogeneous disorder because the variations in a_n, and hence of the local $T_c(H)$, become larger when H increases. As an illustration we consider a long inhomogeneous cylinder $[L\sim 100\ \mu\text{m} \gg \xi(0)]$ of constant section (R =75 nm, t=35 nm) but with a varying diffusivity or equivalently $\xi(0)$ along the axis [see Fig. 1(a)]. The distribution of coherence length is centered around the average value $\simeq$119 nm with the average deviation $\simeq$8 nm. The monotonicity of the spatial variation is not necessary and it is just chosen here to clearly highlight the cause of the broadening. The inhomogeneity of $\xi(0)$ gives rise to a situation in which some of the regions of the sample gradually become superconducting when the temperature is lowered. In the present example the transition starts from the section at $z=L$ and ends at $z=0$ [see Fig. 1(b)]. It is homogeneous only for H =0 (at the mean-field description). As illustrated by Fig. 1(b) the difference of local $T_c(H)$ between the two ends of the cylinder, and thus the temperature width of the resistive transition, gets larger with increasing H and especially when the applied flux approaches half-integer values of Φ_0.

The corresponding dependence of the cylinder resistance on the magnetic field and on the temperature is plotted in Fig. 2. It nicely reproduces the main features of the experimental observations reported by Liu *et al.*[7] for cylinders of the same dimensions (e.g., R=75 nm and t=30 nm for the cylinder of Fig. 1A in Ref. 7). The zero resistance at zero temperature is observed only for an applied flux lower than $1.5\Phi_0$ because the finite thickness of the cylinder wall destroys the periodic behavior predicted for a small thickness. And the resistances at $|\Phi|\simeq 0.5\Phi_0$ and at $2\Phi_0$ are finite fractions of the normal resistance ρ_N instead of being equal to 0 or ρ_N as it would be expected for a homogeneous cylinder. This results from an incomplete transition of the whole cylinder due to the large broadening of the $T_c(H)$ distribution at finite field value.

A good quantitative agreement with experiment can be achieved as illustrated by Fig. 3 where we calculate the temperature dependences of the resistance at different field values for a cylinder with R=131.5 nm and t=30 nm corresponding to sample Al-6 in Ref. 8. We note that the formula, originally derived for a flat film, that is used in Refs. 7 and 8 to calculate the coherence length from the measured upper critical field moderately overestimates it. We find, for example, an average $\xi(0)$ of around 110 nm in sample Al-6 while it is estimated to be 150 nm in Ref. 8. For simplicity we assume here a monotonous decrease in the coherence length along the tube. Such kind of distribution yields a regular resistance variation at the transition without any steplike feature.

B. Steplike features

As shown above a monotonous distribution of parameters along the cylinder cannot produce the steplike features in the resistance curves. However in a long cylinder the variation is unlikely to be monotonous. If the modulations occur on a scale longer than the coherence length, large regions of the cylinder around the local extrema can separately undergo a phase transition at different temperatures and then induce several jumps in the $R(T)$ curves.

We calculate as an example the resistance curves of aluminum cylinder Al-3 from,[8] for which $R\simeq 75$ nm, $t\simeq 33$ nm, and $\Phi=\Phi_0/2$ at $H\simeq 580$ G. We again assume for simplicity that only $\xi(0)$ varies along the axis. Its spatial variations are fitted to the experimental resistance curve at 435 G and then used to compute the temperature dependence

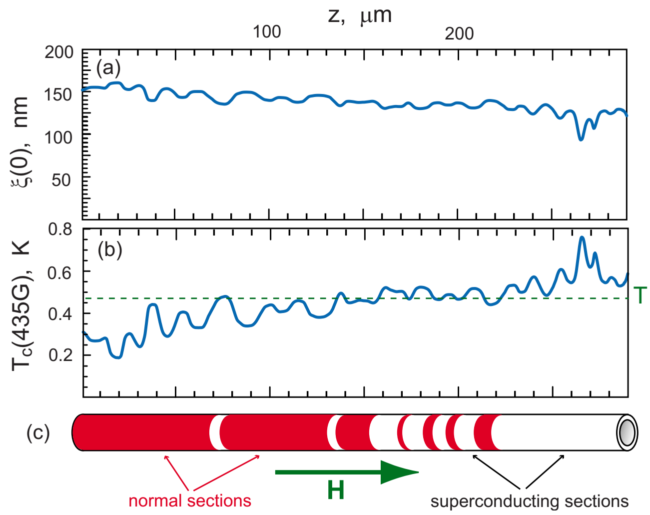

FIG. 4. (Color online) Coordinate dependences (a) of the zero-temperature coherence length $\xi(0)$ and (b) of the local transition temperature $T_c(H)$ at applied field H=435 G, used to fit the resistance curves of sample Al-3 from Ref. 8. (c) Distribution of normal and superconducting sections along the cylinder at the temperature T=0.47 K.

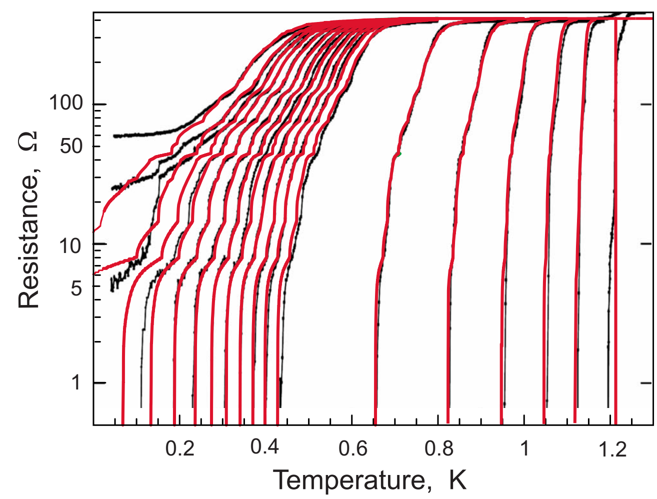

FIG. 5. (Color online) Thick lines: theoretical temperature dependence of the cylinder resistance for applied magnetic field of (from left to right) 456, 449, 445, 440, 435, 430, 425, 420, 415, 410, 405, 400, 347, 295, 246, 197, 145, and 0 G. Joined dots: experimental data from Ref. 8.

of the resistance at all other fields. The inferred distribution of $\xi(0)$ is plotted in Fig. 4. It is determined following three remarks. First, there is a one-to-one correspondence between the coherence length and the critical temperature when other parameters are fixed. So the fraction of the cylinder with a defined value of $\xi(0)$ is in principle known but not the positions where the coherent length has this value. Second, smooth spatial variations in $\xi(0)$ possess local extrema which result in local extrema of $T_c(H,z)$ at fixed field. The sections of the cylinder around these extrema yield divergent contributions $\propto|\partial T_c/\partial z|^{-1}$ to the resistance variations dR/dT and are responsible for the steplike irregularities in the slope of the curves. Finally, the amplitude of the resistance jump at a step is more or less proportional to $|\partial^2 T_c/\partial z^2|^{-1/2}$ at an extremum. These considerations put constraints on the actual distribution of $\xi(0)$ yet different possibilities still exist since the positions of several extrema may indeed be changed without significantly modifying the total resistance (within experimental uncertainties).

Figure 5 shows the calculated curves of resistance for different applied fields. Remarkably, this simple model gives a very good agreement with the experiment.[8] The local definition of the transition temperature is possible, thanks to the slow variations in structural features. This procedure is not justified near $T_c(0)$ where $\xi(T)$ exceeds the length scale of the modulations; however this is compensated by the vanishing dispersion of $T_c(H,z)$ at H=0. The calculations reproduce the resistance nearly for all magnetic fields, except in the vicinity of zero and near the half-flux quantum.

V. DISCUSSIONS AND CONCLUSION

The inferred distribution of $\xi(0)$ that reproduces the experimental resistances is modulated on distances of the order of 10 μm or more. This is larger than the zero-temperature coherence length, the electron mean-free path, or the radius and is consistent with the assumption of smooth variation. The average variance of $\xi(0)$ is around 10%–20% which appears compatible with the inhomogeneity observed among the samples. For instance, in Table I of Ref. 8, a variation in the diameter of sample Al-4 in the range of 157–169 nm is reported. Another illustration of the experimental variability is obtained for the mean-free path by comparing the data for Al-3 and Al-4 in Table I of Ref. 8: being made of the same material and having close diameters these samples differ drastically by their estimated mean-free paths (16 and 6.1 nm, respectively).

Experiments show a narrow transition at zero field but not the vertical drop of resistance predicted by the mean-field theory (see Figs. 3, 5, and 6). This small difference is expected since thermal fluctuations slightly broaden the zero-field transition in low-T_c superconductors. This effect is accentuated if the mean-field $T_c(0)$ varies along the cylinder,[24] which for simplicity we have not assumed in our model but is possible in samples with the cylinder wall so thin that the variations in surface roughness inhomogeneously affect the normal density of states at Fermi level or the electron-phonon coupling to a significant extent. As it is the case for $\xi(0)$ there are likely cylinders where $T_c(0)$ would smoothly vary with position. In such a situation the $R(T)$ curve at zero field could also display several steps. The irregular variation in the observed zero-field resistance in Fig. 5 could be interpreted as such for example (however the small temperature width of the transition and the smearing effect of thermal fluctuations prevent from drawing a definitive conclusion). We note that the inhomogeneous distribution of $T_c(0)$ alone cannot explain the measurements.[7,8] According to equality (9), in the case that $T_c(0)$ varies while $\xi(0)/R$ is constant, the resistive transition would get sharper with increasing applied flux, which is the opposite of the experimental observations.

Several reasons could explain the divergences below 0.2 K observed at high field in Fig. 5. The larger experimental

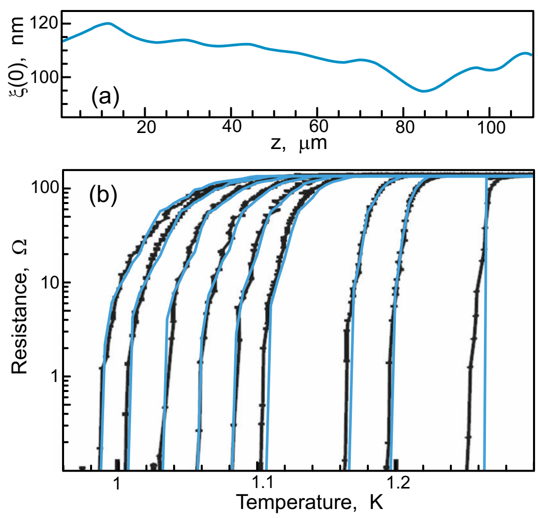

FIG. 6. (Color online) (a) Proposal for an alternative nonmonotonous $\xi(0)$ distribution and (b) the corresponding temperature dependences of the resistance (lines) at different field values, which fit the same experimental data (joined dots) as in Fig. 3.

resistance indicates that certain regions are resistive while according to their calculated transition temperatures they should not. The discrepancy could be explained, for example, by the gapless state which appears just below the critical field.[25] Going beyond the local approximation and taking into account proximity effects should furthermore yield quantitative modifications to the local resistivity. The extra resistance could also be generated at field-weakened superconducting links around local maxima of $\xi(0)$ where the current dynamically generates phase-slip centers.[19] We note by the way that this scenario may also explain why several steplike features are less pronounced or even disappear in measurements at high bias currents.[23] Finally contributions of critical fluctuations in the proximity of the quantum phase transition could also be expected.

As concerns the dependence of the step structure on the dimensions of the cylinder it is naturally explained by our model. Superconductivity is affected by the diffusivity only through its averaged value over an annulus of dimension $\sim\xi(0)$ along the cylinder axis. So for a given radius R the relative variance of the diffusivity behaves as the square root of $1/R$ like the variance of the number of crystal defects in the same averaging volume. Then the relative variance of the local transition temperature at fixed applied flux behaves as $\langle[\delta\xi(0)/R]^2\rangle \sim R^{-5/2}$. This means that the temperature separation between two steps in the resistive transition rapidly decreases with increasing radius. This is fully confirmed by comparing the $R(T)$ curves in Ref. 8 for the samples Al-1 to Al-4 having the diameters in the range of 150–169 nm and Al-6 with $d=263$ nm. The steps are clearly seen for the formers and almost disappear in the latter cylinder. Accordingly sample Al-5 of diameter 212 nm shows an intermediate width of the steps. The step separations become eventually smaller than the experimental precision in cylinders with a large diameter. As an illustration, one can compare the calculated resistances for two alternative proposals of the $\xi(0)$ distribution, a monotonous one in Fig. 3 and a nonmonotonous one in Fig. 6, which fit very well the data for sample Al-6, the cylinder with the largest diameter: the differences between the theoretical curves are not larger than the experimental uncertainty. Besides we note that the shorter a sample is, the less numerous are the extrema present in the variation in $\xi(0)$ and thus the less numerous are the steps in the resistive transition. This also agrees with what is observed for the shortest cylinders Al-5 and Al-6. In the limit where no extremum is present a smaller widening of the transition with no steplike variations should however subsist due to the uncontrollable disorder.

In Ref. 8 it is claimed that the observed steplike features are regular and moreover follow the log 2 slope. A careful examination shows that these statements are not sufficiently supported by experiment. For example, the experimental points for sample Al-1 in Fig. 4(a) of Ref. 8 are located on a strongly curved line, not a straight one (even on a logarithmic plot). Moreover the slopes of the supposed linear lines are not equal for the two chosen tubes. Finally, as we have shown an irregular separation in normal and superconducting regions can explain the resistance behavior very well without relying on such an exotic scenario. As pointed out by the discussion in Refs. 12 and 23, in order to lift any doubt the direct observation of the real distribution of the order parameter along the cylinder is highly desirable. It can be achieved by probing the superconducting and normal sections with scanning tunneling microscopy (STM) for example. If the separation is triggered by an inhomogeneous distribution of diffusivity as it is assumed in our model, it must correlate with the variations in local resistivity in the normal state. Since the alternating regions have typical lengths of several micrometers, low temperature scanning laser microscopy[26,27] could also be conveniently used. As an alternative to STM, this method has the advantage of its setup simplicity and its independence from the surface quality of the sample. Besides the experimental observation of the long-range modulation of disorder, it would also be interesting to fabricate and investigate tubes of thickness $\sim\lambda^2/R$ in order to probe the other scenario discussed in Sec. II, in which the inhomogeneous distribution of the order parameter is induced by a first-order phase transition in the applied field.

In conclusion, we determined a criterion for first-order phase transition for hollow nanocylinders in magnetic field. According to it, the recently studied superconducting cylinders cannot exhibit a spontaneous transition to a symmetry-broken order parameter. Instead, we found that the broadening of the resistive transition with increasing field is due to the partial destruction of superconductivity triggered by the structural inhomogeneities of the tubes. The long-range modulated disorder naturally explains the steplike features in the temperature dependence of the resistance as the manifestation of local phase transitions around the extrema in its spatial distribution. The local mean-field calculations reproduce quantitatively the experimental data for nearly all fields and temperatures, which restrict the potential effects of quantum critical fluctuations to the close vicinity of the zero-temperature phase transition.

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